E-Supplement for Chapter 12 LP Solver

Linear Programming with Excel Solver®

Problem statement

A company produces three products: A, B, and C. The selling prices are $10, $20, and $30 respectively. Variable production costs are $3, $12, and $25 respectively. Demand for A is between 30 and 100 units, Demand for B is between 20 and 70 units, and demand for C is between 15 and 50 units. All the products need to be produced at three machines M1, M2, and M3 with total capacity 300, 250, and 203 capacity units respectively. Production of the product A seizes 2 capacity units at M1, one capacity unit at M2, and 2 capacity units at M3. Production of the product B seizes 3 capacity units at M1, 4 capacity units at M2, and 2 capacity units at M3. Production of the product C seizes 4 capacity units at M1, 5 capacity units at M2, and 2 capacity units at M3. Currently, the company produces 50 units of A, 25 units of B, and 12 units of C.

Is the decision to produce 50 units of A, 25 units of B, and 12 units of C optimal in regard to profit maximization? If yes, why? If not, what is optimal decision?

→ In order to answer this question, we formulate a linear programming (LP) optimization model for the management problem considered above with the help of Excel Solver® that needs to be activated and is placed in the "Data" in the Analysis group on the Data tab.

LP model in Excel Solver®

In the first step, we setup the initial data about prices, costs, demand, and capacity constraints. Second, the objective function and the solution area need to be defined (Fig. 1).

Data:	A	B	C				
Price	10	20	30				
VarCosts	3	12	25				
Profit	7	8	5				
Min Demand	30	20	15				
Max Demand	100	70	50			**Solution:**	
				Capacity		Capacity _()	Utilization
M1	2	3	4	300			
M2	1	4	5	250			
M3	2	2	2	203			
Z=	**0**						
Solution:	A	B	C				
Production							

Fig. 1. Problem statement for solution in Excel Solver®

In Fig. 2, the definition of the objective function, decision variables, and constraints is shown.

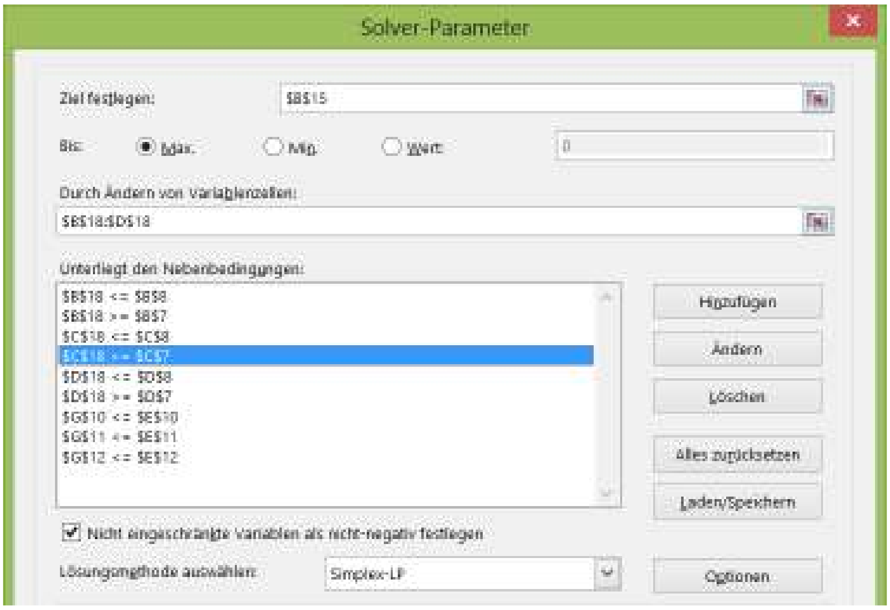

Fig. 2. Setting the model in Excel Solver®

Finally, the solution is created with the help of, e.g., Simplex-LP by pressing the button “Solve”. The results are presented in Fig. 3.

Data:	A	B	C				
Price	10	20	30				
VarCosts	3	12	25				
Profit	7	8	5				
Min Demand	30	20	15				
Max Demand	100	70	50			Solution:	
				Capacity		Capacity _()	Utilization
M1	2	3	4	300		262,5	87,5
M2	1	4	5	250		250	100
M3	2	2	2	203		203	100
Z=	710						
Solution:	A	B	C				
Production	57	30	15				

Fig. 3. Solution to the model

According to the suggested solution, optimal decision in regard to maximal profit is to produce 57 units of A, 29 units of B, and 15 units of C. In this setting, the capacity at M2 and M3 would be utilized to 100%, and M2 capacity would have utilization of 87.5%. This decision would allow us to get a profit of $710. If setting the initial production quantities (50 units of A, 25 units of B, and 12 units of C) into the objective function, we get total profit of $610. As such, the usage of LP optimization allows increasing the profit by 16.4%.

Sensitivity analysis

After pressing the „Solve" button, the option "*Sensitivity*" is displayed. Let us analyse sensitivity of our solution (Fig. 4).

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B18	Production A	57	0	7	1	5
C18	Production B	29,5	0	8	20	1
D18	Production C	15	-3,33333	5	3,33333	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
G10	M1 Capacity _()	262,5	0	300	1E+30	37,5
G11	M2 Capacity _()	250	0,333333	250	81	28,5
G12	M3 Capacity _()	203	3,333333	203	45	40,5

Fig. 4. Sensitivity analysis results

In the upper part “Variable Cells” of the sensitivity report, we can observe optimal production quantities for the products A, B, and C in the column “Final Value”. Next column “Reduced costs” reports on possible profit increases (or decreases, as in our case in Fig. 4) in the case of production quantity increase of each additional unit of the respective products. The last two columns “Allowable Increase” and “Allowable Decrease” determine the production quantitiy change range for which the “Reduced Costs” values hold true. For example, the reduced costs = 0 holds true for the production quantity range [52; 58] of product A.

In the part “Constraints” of the sensitivity report, we can observe computed total capacities of machines M1, M2, and M3 in the column “Final Value”. Next column “Shadow Price” reports on possible profit decreases in the case of capacity increase of each additional unit at the respective machine. For example, increase of M3 capacity from 203 units to 204 units would increase our total profit by $3.33. The last two columns “Allowable Increase” and “Allowable Decrease” determine the capacity change range for which the “Shadow Price” values hold true. For example, the shadow price = 0.33 for M2 holds true for the capacity range [221.5; 331] at machine M2.

Capacity extension analysis

The management of the company analysed the sensitivity report results and observed that an increase in M3 capacity may potentially provide higher profit increase. M3 capacity extension from 203 units to 250 units requires an investment of $100. Is it sensible to invest?

→ We solve the LP model for new M3 capacity = 250. The result is shown in Fig. 5.

Data:	A	B	C				
Price	10	20	30				
VarCosts	3	12	25				
Profit	7	8	5				
Min Demand	30	20	15				
Max Demand	100	70	50			**Solution:**	
				Capacity		Capacity _()	Utilization
M1	2	3	4	300		300	100
M2	1	4	5	250		245	98
M3	2	2	2	250		250	100
Z=	**865**						
Solution:	A	B	C				
Production	90	20	15				

Fig. 5. Optimal solution for extended M3 capacity

It can be observed from Fig. 5 that total profit increases from $710 to $865. Even if the capacity extension needs an investment of $100, it allows inctreasing the profit by $155. The sensitivity report for this solution is depicted in Fig. 6.

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B18	Production A	90	0	7	1	1,666666667
C18	Production B	20	0	8	2,5	1
D18	Production C	15	-4	5	4	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
G10	M1 Capacity _()	300	1	300	1,666666667	0
G11	M2 Capacity _()	245	0	250	1E+30	5
G12	M3 Capacity _()	250	2,5	250	0	2

Fig. 6. Sensitivity report for optimal solution with extended capacity of M3

Demand fulfillment analysis

From Figs 5 and 6, the sales manager can still observe that demand for products B and C is much higher as our production. The question is: what capacity do we need to cover maximum demand for all the products? One possible option to answer this question is to solve the model against almost unlimited capacity, say 1,000 capacity units at each machine. The results are presented in Fig. 7.

Data:	A	B	C				
Price	10	20	30				
VarCosts	3	12	25				
Profit	7	8	5				
Min Demand	30	20	15				
Max Demand	100	70	50			**Solution:**	
				Capacity		Capacity _()	Utilization
M1	2	3	4	1000		610	61
M2	1	4	5	1000		630	63
M3	2	2	2	1000		440	44
Z=	**1510**						
Solution:	A	B	C				
Production	100	70	50				

Fig. 7. Optimal solution for "dummy" unlimited capacity

It can be observed from Fig. 7 that we need 610 capacity units at M1, 630 – at M2, and 440 at M3 in order to satisfy maximum demand. This would result in total profit of $1,510.

Finally, a limits report can be created similar to sensitivity report (Fig. 8).

Cell	Target Name	Value
B15	Z= A	1510

Cell	Adjustable Name	Value	Lower Limit	Target Result	Upper Limit	Target Result
B18	Production A	100	30	1020	100	1510
C18	Production B	70	20	1110	70	1510
D18	Production C	50	15	1335	50	1510

Fig. 8. Limits report

In the columns “Target results”, the limits report depicts the values of the objective function for lower and upper limits of the demand constraints. For example, it can be observed from Fig. 8 that in the case of production quantity reduction for product A from 100 to 30 units, total profit will be reduced from $1,510 to $1,020. We can also observe that the lowest perofrmance impact would have the reduction in product C production quantities.

Now it is the task of COO and CFO to find a compromise on investements in new capacity and production profitability. For such an executive meeting, marketing, sales, procurement and supply chain managers need to be invited in order to take into accout future demand developments, supplier capacities, and supply chain integration in the case of drastically increased capacities and production quantities.